MATHEMATICS SOLUTION
(CBCGS SEM – 4 MAY 2018)
BRANCH – IT ENGINEERING

1a) A discrete random variable has the probability distributing given below: (5)

X	-2	-1	0	1	2	3
P (x)	0.2	k	0.1	2k	0.1	2k

Find k, the mean and variance.

Ans. For any probability mass function, $\sum_{i=-\infty}^{\infty} p_i = 1$

$\therefore$ P(X = -2) + p(x = -1) + p(x = 0)+p(X = 1) + p(X =2) + p(x = 3) = 1

$\therefore 0.2 + k + 0.1 + 2k + 0.1 + 2k = 1$

$\therefore 5k + 0.4 = 1$

$\therefore 5\,k = 0.6$

$\therefore k = 0.12$

$\therefore$ The probability distribution P(X) of X is

X	P(X)	P_iX_i	$P_iX_i^2$
-2	0.2	-0.4	0.80
-1	K = 0.12	-0.12	0.12
0	0.1	0.00	0.00
1	2k = 0.24	0.24	0.24
2	0.1	0.20	0.40
3	2k = 0.24	0.72	2.16
	Total	0.64	3.72

Mean = E (X) = $\Sigma p_i X_i$ = 0.64

$E(X^2) = \Sigma\, P_i X_i^{\,2} = 3.72$

Variance = $E(X^2) - [E(X)]^2$

$= 3.72 - 0.64^2$

$= 3.3104$

$\therefore$ K = 0.12; Mean = −0.64; Variance = −3.3104.

1b) Find smallest positive integer modulo 5, to which $3^2.3^3.\ 3^4.3^{10}$ is congruent. **(5)**

Ans. $3^2.3^3.\ 3^4.3^{10} = 3^{2+3+4+10} = 3^{19}$

We know, 9=2×5+(-1)

$\therefore\ 9 \equiv -1 (\text{mod} 5)$

$\therefore 3^2 \equiv (-1)\ (\text{mod } 5)$

$\therefore (3^2)^9 \equiv (-1)^9\ (\text{mod} 5)$ {∵If $a \equiv b (\text{mod } m)$ then $a^k \equiv b^k (\text{mod } m)$}

$\therefore 3^{18} \equiv -1 (\text{mod} 5) \longrightarrow (1)$

We know, if $a \equiv b$ (modm) and c is any integer, then $ac \equiv b\,c$ (modm)}

$\therefore$ From (1), $3^{18} \times\ 3 \equiv -1 \times 3 (\text{mod} 5)$

$\therefore\ 3^{19} \equiv -3 (\text{mod} 5)$

$\therefore\ 3^{19} \equiv 2 (\text{mod} 5)$

Hence, smallest positive integer modulo 5, to which $3^2.3^3.3^4.3^{10}$ is congruent is 2

1c) Regression lines are given by 6y = 5x + 90, 15x = 8y+130 , σ_x^2 = 16. **(5)**
Find mean for x and y, correlation coefficient between x and y, and σ_y^2.

Ans. 6y = 5x + 90

$\therefore y = \frac{5}{6}x + \frac{90}{6}$

$\therefore y = \frac{5}{6}\ x + 15 \quad \longrightarrow (1)$

And, $15x = 8y + 13$

$\therefore y = \frac{15}{8}x - \frac{130}{8} \longrightarrow (2)$

Let $b_1 = \frac{5}{6}$ and $b_2 = \frac{15}{8}$

Since $|b_1| < |b_2|$,

$$b_{yx} = b_1 = \frac{5}{6}\ \&\ b_{xy} = \frac{1}{b_2} = \frac{8}{15} \ \rightarrow (3)$$

∴ Equation (1) is regression equation of Y and X type and equation (2) is regression equation of X and Y type.

From (1) and (2)

$\frac{5}{6}x + 15 = \frac{15}{8}x - \frac{65}{4}$

$\therefore \frac{65}{4} + 15 = \frac{15}{8}x - \frac{5}{6}x$

$\therefore \frac{125}{4} = \frac{25}{24}x$

$\therefore x = 30$

Substitute x = 30 in (1)

$\therefore y = \frac{5}{6}(30) + 15 = 40$

Now, $r = \pm\sqrt{b_{yx} . b_{xy}}$

$= \pm\sqrt{\frac{5}{6} x \frac{8}{15}}$ (from 3)

$= \pm\frac{2}{3}$

Since, b_{yx} and b_{xy} are both positive, 'r' is positive.

$\therefore r = \frac{2}{3} \longrightarrow (4)$

Also, given, ${\sigma_x}^2 = 16 \quad \therefore \sigma_x = 4 \quad \longrightarrow (5)$

Using, $b_{yx} = r\frac{\sigma_y}{\sigma_x}$

$\therefore \frac{5}{6} = \frac{2}{3}.\frac{\sigma_y}{\sigma_x}$ (From 3, 4 & 5)

$\therefore \sigma_y = 5$

$\therefore {\sigma_y}^2 = 25$

Ans. 1) $\bar{x} = 30; \bar{y} = 40;$

2) $r = \frac{2}{3};$

3) ${\sigma_y}^2 = 25$

1d) Show that G = {1,-1,i,-i} is a group under multiplication of complex number. (5)

Ans. Let a, b ∈ G

The composition table is

*	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

From above table, we observe,

a* b exists and a * b∈ G.

∵ *is binary operator in G.

G1:

Multiplication of complex number is associative.

∴ * is associative.

G2:

From table, we observe, first row is same as the header.

∴ 1∈ G is the identity.

∴ Identity exists.

G3:

From table, we observe, identity elements (i.e.1)is present in each row.

∴ $1^{-1} = 1$; $(-1)^{-1} = -1$; $i^{-1} = -i$; $(-i)^{-1} = i$

∴inverse of each elements exist and each inverse ∈ G.

∴ Inverse exists.

Hence, G is group usual multiplication of complex number.

2a) Show that $111^{333} + 333^{111}$ is divisible by 7. **(6)**

Ans.

We know , $111 = 16 \times 7 + (-1)$

$\therefore 111 \equiv -1 \pmod 7$

$\therefore 111^{333} \equiv (-1)^{333} \pmod 7$

$\therefore 111^{333} \equiv -1 \pmod 7 \longrightarrow (1)$

Similarly, $333 = 47 \times 7 + (4)$

$\therefore 333 \equiv 4 \pmod 7$

$\therefore\ 333^3 \equiv 4^3 \pmod 7$

$333^3 \equiv 64 \pmod 7$

$333^3 \equiv 1 \pmod 7$

$(333^3)^{37} \equiv 1^{37} \pmod 7$

$(333)^{111} \equiv 1 \pmod 7 \longrightarrow (2)$

$\therefore$ Adding (1) & (2), $111^{333} + 3333^{111} \equiv 1 \pmod 7 + (-1) \pmod 7$

$\therefore 111^{333} + 333^{111} \equiv (1-1) \pmod 7$

$\therefore\ 111^{333} + 333^{111} \equiv 0 \pmod 7$

i.e, Remainder = 0 when $111^{333} + 333^{111}$ is divided by 7.

$\therefore\ 111^{222} + 333^{111}$ is divisible by 7.

2b) The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distribution over a week. **(6)**

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of Accidents	13	15	9	11	12	10	14	84

Ans. n = 7

Total accidents = 84

If equally distributed, expected accidents per day $= \frac{84}{7} = 12$

Since frequency for Tuesday is less than 10, we combine it with that for Wednesday.

	Observed Frequency (0)	Expected Frequency (E)	$x^2 = \frac{(o-E)^2}{E}$
Sun	13	12	0.0833
Mon	15	12	0.7500
Tue Wed	$\left.\begin{matrix}11\\9\end{matrix}\right\}= 20$	$\left.\begin{matrix}12\\12\end{matrix}\right\} =24$	0.6667
Thur	12	12	0.0000
Fri	10	12	0.3333
Sat	14	12	0.3333
		Total	2.1667

Step 1:

Null Hypothesis (H_0) : Accident are uniformly distributed over the week.

Alternative Hypothesis (H_0) : Accidents are not uniformly distributed over the week.

Step 2:

Level of significance (LOS) = 5%

Degree of freedom = (n-1) – 1 = 7-1 -1 = 5.

(Due to combining, the effective DOF decreases by 1)

∴ Critical value $(x_a{}^2) = 11.0705$

Step 3:

$$x_{cal}{}^2 = \Sigma \frac{(O-E)^2}{E} = 2.1667$$

Step 4:

Since $x_{cal}{}^2 < x_a{}^2$, H_0 is accepted.

∴ Accidents are uniformly distributed over the week.

2c) write the following permutation as the product of disjoint cycle. (4)

f = (1 3 2 5)(1 4 5)(2 5 1)

Ans.

$f(1) = (1\ 3\ 2\ 5)(1\ 4\ 5)\ (2\ 5\ 1)\ \ (1)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5\ )\ (2)$

$= (1\ 3\ 2\ 5)\ (2)$

$\therefore f(1) = 5$

$f(2) = (1\ 3\ 2\ 5)(1\ 4\ 5)\ (2\ 5\ 1)\ \ (2)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (5)$

$= (1\ 3\ 2\ 5)\ (1)$

$\therefore f(2) = 3$

$f(3) = (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (2\ 5\ 1)\ \ (3)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (3)$

$= (1\ 3\ 2\ 5)\ (3)$

$\therefore f(3) = 2$

$f(4) = (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (2\ 5\ 1)\ \ (4)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (4)$

$= (1\ 3\ 2\ 5\ )\ (5)$

$\therefore f(4) = 1$

$f(5) = (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (2\ 5\ 1)\ \ (5)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (1)$

$= (1\ 3\ 2\ 5)(4)$

$\therefore f(5) = 4$

$\therefore f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 1 & 4 \end{pmatrix}$

Hence, expressing permutation f as the product of disjoint cycle we have

$f = (1\ 5\ 4)\ (2\ 3\ )$

2d) Simplify as sum of product (A+B) (A +B')(A'+B)(A'+B') **(4)**

Ans. Consider,

(A+B) (A +B') (A'+B)(A'+B')

≡[(A +B)(A'+B')][(A + B') (A'+ B)]

≡[A (A' +B')+B(A'+B')][A (A'+ B)+B'(A'+ B)]

≡[(AA'+AB'+BA'+BB'][AA'+AB+B'A+B'B]

≡ [0+AB'+BA'+0][0+AB+B'A'+0] (Complement Law)

≡ [AB'+BA'][AB+B'A'] (Identify law)

≡ AB' [AB +B'A'] + BA' [AB +B'A']

≡ (AB')(AB) + (AB')(B'A')+(BA')(AB)+(BA')(B'A')

≡ (AB')(BA)+(B'A)(A'B')+(BA')(AB)+(A'B)(B'A') (Complement Law)

≡ A (B'B)A+B'(AA')B'+B(A'A)B+A'(BB')A' (Associative Law)

≡A(0)A+B'(0)B'+B(0)B+A'(0)A' (Complement Law)

≡ 0 + 0+0+0 (Domination Law)

≡ 0 (idempotent Law)

∴ (A +B)(A+B')(A'+B)(A'+B') = 0

3a) Find gcd (2378 , 1769) using Euclidean Algorithm. **(6)**
Also find x and y such that 2378x +1763y = gcd (2378,1769)

Ans. **Part I:**

Part I: Let a = 1769 and b = 2378 , Using Euclid Algorithm,

1	2378 = 1×1769 + 609	∴ 609 = b – a
2	1769 = 2 × 609 +551	∴ 551 = a - 2(b-a) ∴ 551 = 3a – 2b
3	609 = 1 × 551 + 58	58 = (b – a) – (3a – 2b) ∴ 58 = 3b – 4a
4	551 = 9×58 + 29	29 = (3a – 2b) – 9(3b – 4a) ∴ 29 = 39a – 29b ⟶(1)
5	58 = 2 × 29+0	

∴gcd (2378,1769) = 29

Part II:

from (1), x = 39 and y = -29

i.e., x_0 = 39 and y_0 = -29 is one solution of 2378x +1763y = gcd (2378,1769)

other solutions are $x = x_0 + \left[\frac{b}{d}\right]$t and y = $y_0 - \left[\frac{a}{d}\right]$t where 't' is arbitrary & d = gcd of a & b i.e. d = (a,b)= 29

$$x = 39 + \left(\frac{2378}{29}\right)t \text{ and } y = -29 - \left(\frac{1769}{29}\right)t$$

∴ other solutions are $x = 39 + 82t$ and y=-–29 – $61t$

3b) Give an example of a graph which has **(6)**

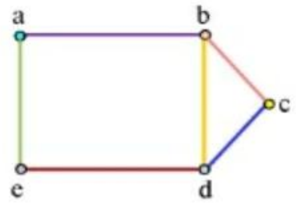
(i) Eulerian circuit but not a Hamiltonian circuit
(ii) Hamiltonian circuit but not an Eulerian circuit
(iii) Both Hamiltonian circuit and Eulerian circuit.

Ans:

(i) Eulerian circuit but not a Hamiltonian circuit

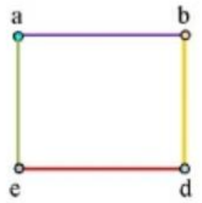
All the vertices are of even degree. Hence by theorem there is Eulerian circuit.
Eulerian circuit : abcdeca
The circuit is not Hamiltonian because there is no circuit which contains all the vertices only once.

(ii) Hamiltonian circuit but not an Eulerian circuit



All the vertices can be traversed only once. Hence there is Hamiltonian circuit.
Hamiltonian circuit : abcdea
The degree of vertices b & d are odd. Hence there is no Eulerian circuit.

(iii) Both Hamiltonian circuit and Eulerian circuit.



Degree of all vertices are even. Hence there is an Euler circuit abdea.

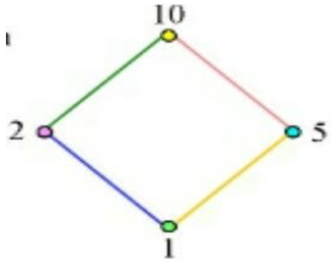
All the vertices can be traversed only once. Hence there is Hamiltonian circuit.
Hamiltonian circuit : abdea

3c) Show that (D_{10},≤) is a lattice.Draw its Hasse diagram. **(8)**

Ans. D_{10} means divisors of 10.

$D_{10} = \{1, 2, 5, 10\}$

The Hasse diagram of R is as shown



We know the relation of divisibility is a partial order relation.

∴Set (D_{10} ,≤)is a poset.

∨	1	2	5	10
1	1	2	5	10
2	2	2	10	10
5	5	10	5	10
10	10	10	10	10

∧	1	2	5	10
1	1	1	1	1
2	1	2	1	2
5	1	1	5	5
10	1	2	5	10

From the two table, we observe, that every pair of elements of D_{10}has a LUB (least upper bound) and GLB (greater lower bound).

Also, each LUB and GLB ∈ D_{10}

Hence, (D_{10},≤) is a lattice.

4a) Calculate the correlation coefficient from the following data: (6)

X	**23**	**27**	**28**	**29**	**30**	**31**	**33**	**35**	**36**	**39**
y	**18**	**22**	**23**	**24**	**25**	**26**	**28**	**29**	**30**	**32**

Ans. Karl Pearson's correlation co-efficient:

Let a = 30 and b = 25

X	Y	U=X-30	V = Y-25	U^2	V^2	UV
23	18	-7	-7	49	49	49
27	22	-3	-3	9	9	9
28	23	-2	-2	4	4	4
29	24	-1	-1	1	1	1
30	25	0	0	0	0	0
31	26	1	1	1	1	1
33	28	3	3	9	9	9
35	29	5	4	25	16	20
36	30	6	5	36	25	30
39	32	9	7	49	49	63
	Total	11	7	215	163	186

n= 10

Karl Pearson's correlation co-efficient is given by,

$$r_{x,y} = r_{u,v} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$= \frac{10(186) - 11 \times 7}{\sqrt{10(215) - 11^2}\sqrt{10(163) - 7^2}}$$

$$= \frac{1783}{\sqrt{2029}\ \sqrt{1581}}$$

$= 0.9955$

$\therefore$ Correlation coefficient $= 0.9955$

4b) let G be a group of all permutations of degree 3 on 3 symbols 1,2&3. Let H={(I(1,2)} be a subgroup of G.
Find all the distinct left cosets of H in G and hence index of H. (6)

Ans. G be a group of all permutations of degree 3 on 3 symbols 1,2 & 3.
$\therefore$ Order of G $= |G| = 3! = 6$

Given, H $= \{I,(1,2)\}$ is a subgroup of G.

$\therefore$ Order of H $= |H| = 2! = 2$

By Lagrange's Theorem, index = Number of different left cosets of subgroup

H$= \frac{|G|}{|H|} = \frac{6}{2} = 3$

Consider, left coset of

$(13)H = (13)\{I,(1\ 2)\} = \{(13)I,(13)(12)\}$

Now,

$(13)I = (13)$(since, I is the identity)

$(13)(12) = (123)$

$\therefore (13)\,H = \{(13),(123)\}$

Similarly, $(23)H = (23)\{I,(1\ 2)\} = \{(23)I,(23)(12)\}$

Now, $(23)I = (23)$ (since, I is the identity)

$(23)(12) = (132)$

$\therefore (23)H = \{(23),(132)\}$

Hence, the three distinct cosets of H are H, (13)H, (23)H , Index = 3

4c) The average marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether boys perform better than the girls. (z_a=2.326). **(4)**

Ans. $n_1 = 32$ and $n_2 = 36$ (>30, it is large sample)

$\overline{x_1} = 72; \overline{x_2} = 70$; $s_1 = 8$; $s_2 = 8$

Step 1:

Null Hypothesis (H_0) :$\mu_1 = \mu_2$ (i.e, performance of boys and girls is equal)

Alternative Hypothesis (H_a) :$\mu_1 > \mu_2$ (i.e, boys perform better than the girls) (one tailed test)

Step 2:

Level of Significance (LOS) = 1%(Two tailed test)

∴ LOS = 2% (one tailed test)

∴ Critical value (z_a)=2.33

Step 3 :

Since sample are large, S.E, $= \sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}$

$= \sqrt{\frac{8^2}{32} + \frac{6^{\ 2}}{36}}$

= 1.732

Step 4 : Test statistic

$z_{cal} = \frac{\overline{x_1} - \overline{x_2}}{S.E.}$

$= \frac{72-70}{1.732}$

= 1.1547

Step 5: Decision

Since$|z_{cal}| < z_a$, H_0 is accepted.

∴ Boys do not perform better than the girls.

4d) A random sample of 15 items gives the mean 6.2 and variance 10.24. can it be regarded as drawn from a normal population with mean 5.4 at 5%LOS ? (4)

Ans. n = 15 (<30, so it is a small sample)

Step 1:

Null Hypothesis (H_0) :$\mu_1 = 5.4$ (i.e, the sample is drawn from a normal population with mean 5.4)

Alternative Hypothesis (H_a) :$\mu_1 \neq 5.4$ (i.e, the sample is not drawn from a normal population with mean 5.4) (Two tailed test)

Step 2:

Level of significance (LOS) = 5% (Two tailed test)

Degree of Freedom = n – 1 = 15 – 1 = 14

$\therefore$ Critical value $(t_a) = 2.145$

Step 3:

Given, sample mean $\bar{x} = 6.2$ and sample variance $s^2 = 10.24$

$\therefore s = \sqrt{10.24} = 3.2$

Standard Error S.E $= \frac{s}{\sqrt{n-1}}$

$= \frac{3.2}{\sqrt{15-1}}$

$= 0.8552$

Step 4: Test Statistic

$t_{cal} = \frac{\bar{x}-\mu}{S.E}$

$= \frac{6.2-5.4}{0.8552}$

$= 0.9354$

Step 5 : Decision

Since $|t_{cal}| < t_a$, H_0 is accepted.

$\therefore$ The sample is drawn from a normal population with mean 5.4.

5a) Derive mgf of Binomial distribution and hence find its mean and variance. (6)

Ans. For Binomial distribution,

$P(X = x) = {}^{n}C_x p^x q^{n-x} \longrightarrow (1)$

By definition, moment generating function about origin $M_0(t) = E(e^{tx})$

$= \sum_{x=0}^{n} p_i\, e^{tx}$

$= \sum_{x=0}^{n} {}^{n}C_x p^x q^{n-x} e^{tx}$ (From 1)

$= \sum_{x=0}^{n} {}^{n}C_x (pe^t)^x q^{n-x}$

$= (q + pe^t)^n [\because \sum_{x=0}^{n} {}^{n}C_x a^x b^{n-x} = (a+b)^n]$

Now, rthmoment $\mu_r' = \left[\frac{d^r}{dt^r} M_0(t)\right]_{t=0} \longrightarrow (2)$

$\therefore \mu_1' = \left[\frac{d}{dt} M_0(t)\right]_{t=0}$

$= \left[\frac{d}{dt}(q + pe^t)^n\right]_{t=0}$

$= [n(q + pe^t)^{n-1}.pe^t]_{t=0}$

$= [n(q + pe^0)^{n-1} pe^0]$

$= [n(q + p)^{n-1}\, p]$

$= [n.1^{n-1}.p]$ ($\because q + p = 1$)

$= n\,p$

Similarly, from (2), $\mu_2' = \left[\frac{d^2}{dt} M_0(t)\right]_{t=0}$

$= \left[\frac{d^2}{dt^2}(q + pe^t)^n\right]_{t=0}$

$= \left[\frac{d}{dt} np(q + pe^t)^{n-1} e^t\right]_{t=0}$

$= [np\,\{(q + pe^t)^{n-1}.e^t + e^t.(n-1)(q + pe^t)^{n-2}\, pe^t\}]_{t=0}$

$= n\,p\,\{ (q + p.1)^{n-1}.1 + 1.(n-1)(q + p.1)^{n-2} p.1 \}$

$= n\,p\,\{(1)^{n-1} + (n-1)(1)^{n-2}\, p\}$ ($\because q + p = 1$)

$= n\,p\,\{1 + np - p\}$

$= n\,p\,\{q + np\}$ ($\because 1 - p = q$)

$= n\,p\,q + n^2p^2$

$\therefore$ Mean $= \mu_1{}' = n\,p$

$\therefore$ Variance $= \mu_2 = \mu_2{}' - \mu_1{}^2$

$= (n\,p\,q + n^2p^2) - (n\,p\,)^2$

$= n\,p\,q$

Hence, for Binomial distribution , Moment Generating Function $= (q + pe^t\,)^n$

Mean $= n\,p$

Variance $= n\,p\,q$

5b) It was found that the burning life of electric bulbs of a particular brand was normally distributed with the mean 1200 hrs and the S.D. of 90 hours. Estimate the number of bulbs in a lot of 2500 bulbs having the burning life: (6)

(i) more than 1300 hours

(ii) between 1050 and 1400 hours.

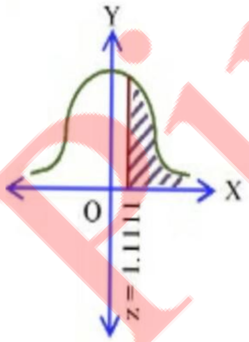
Ans. Mean (m) = 1200

Standard deviation (σ) = 90

N = 2500

Let x denote the burning life of the electric bulb.

(i) P (burning life more than 1300 hours) = P (X > 1300)



$= \ P\left(\frac{x-m}{\sigma} > \frac{1300-1200}{90}\right)$

$= \ P(\,z > 1.1111)$

$= 0.5$ – Area between ‘z = 0’ to ‘z = 1.1111’

$= 0.5 - 0.3667$

= 0.1333

∴ Number of bulbs with burning life more than 1300 hours =N × P (x>1300)

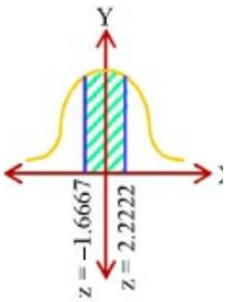
= 2500 x 0.1333

= 333.15

≈ 333 bulbs

(ii) P(burning life between 1050 and 1400 hours)= P(1050< X <1400)

$= P\left(\frac{1050-1200}{90} < \frac{X-m}{\sigma} < \frac{1400-1200}{90}\right)$



= P (-1.6667<z<2.2222)

= Area between ‘z = 0’ to ‘z=-1.6667’ + Area between ‘z = 0’ to ‘z=2.2222’

= 0.4522 + 0.4869

= 0.9391

∴ Number of bulbs with burning life between 1050 and 1400 hours

= N ×P(1050<X<1400)

= 2500 x 0.9391

= 2347.7

≈ 2348 bulbs

Number of bulbs with burning life more than 1300 hours = 333bulbs

Number of bulbs with burning life between 1050 and 1400 hours = 2348 bulbs

5c) Find inverse of 8^{-1}(mod77) using Euler's theorem. **(4)**

Ans.

Euler's Theorem: $= a^{-1} (\text{mod } m) \equiv a^{\phi(m)-1} (\text{mod } m)$

Here, a = 8 and m = 77

$8^{-1} (\text{mod } 77) \equiv 8^{\phi(77)-1} (\text{mod } 77) \longrightarrow (1)$

Now $\Phi(77) = \Phi(11 x 7)$

$= (11-1)(7-1)$

$= 60$

$\therefore$ *eq* 1 *becomes* $8^{-1} (\text{mod } 77) \equiv 8^{60-1} (\text{mod } 77)$

$\equiv 8^{59} (\text{mod } 77)$

$\equiv (8^5)^{11} . 8^4 \ (\text{mod } 77)$

$\equiv (43)^{11} . 8^4 \ (\text{mod } 77)$

$\equiv (43^2)^5 . 43 . 8^4 \ (\text{mod } 77)$

$\equiv (1)^5 . 43 . 15 \ (\text{mod } 77)$

$\equiv 29 \ (\text{mod } 77)$

5d) Find the Jacob's symbols of $\left(\frac{32}{15}\right)$. **(4)**

Ans. $\left(\frac{32}{15}\right) = \left(\frac{32}{5X\ 3}\right) = \left(\frac{32}{5}\right)\left(\frac{32}{3}\right) \longrightarrow (1)$

But,

$32 = 5 \times 6 + 2 \ \therefore \left(\frac{32}{5}\right) = \left(\frac{2}{5}\right) \longrightarrow (2)$ and,

$32 = 3 \times 10 + 2 \ \therefore \left(\frac{32}{5}\right) = \left(\frac{2}{3}\right) \longrightarrow (3)$

$\therefore$ From (1), (2) & (3), $\left(\frac{32}{15}\right) = \left(\frac{2}{5}\right)\left(\frac{2}{3}\right) \longrightarrow (4)$

We know, for odd prime p, $\left(\frac{2}{p}\right) = (-1)^{p^2-1/8}$

When p = 3, $\left(\frac{2}{3}\right) = (-1)^{3^2-1/8} = (-1)^1 = -1 \ \longrightarrow (5)$

When p = 5, $\left(\frac{2}{5}\right) = (-1)^{5^2-1/8} = (-1)^3 = -1 \longrightarrow (6)$

$\therefore$ From (1),(2) & (3), $\left(\frac{32}{15}\right) = (-1)(-1)$

$\therefore \left(\frac{32}{15}\right) = 1$

6a) Solve : x =1(mod3) , x= 2(mod5) , x=3(mod7) **(6)**

Ans:

$a_1 = 1 \qquad m_1 = 3$
$a_2 = 2 \qquad m_2 = 5$
$a_3 = 3 \qquad m_3 = 7$

$M = m_1.m_2.m_3$

$M = 3 \times 5 \times 7 = 105$

$M_1 = 5 \times 7 = 35$

$M_2 = 3 \times 7 = 21$

$M_3 = 3 \times 5 = 15$

$M_1x \equiv 1(mod\ m_1)$
$35x \equiv 1(mod\ 3)$
$1 \equiv 35x\ (mod\ 3)$
$1 \equiv 2x(mod3)$
$1 \equiv -x(mod\ 3)$
$x \equiv -1(mod\ 3)$
$x \equiv 2(mod\ 3)$

$M_2x \equiv 1(mod\ m_2)$
$21x \equiv 1(mod\ 5)$
$1 \equiv 21x\ (mod\ 5)$
$1 \equiv 1x(mod\ 5)$
$x \equiv 1(mod\ 5)$

$M_3x \equiv 1(mod\ m_3)$
$15x \equiv 1(mod\ 7)$
$1 \equiv 15x(mod\ 7)$
$1 \equiv 1x\ (mod\ 7)$
$x \equiv 1(mod\ 7)$

By Chinese Remainder Theorem,

$\therefore \quad x \equiv [a_1M_1x_1 + a_2M_2x_2 + a_3M_3x_3]\ modulo\ \ M.$

$x \equiv [(1)(35)(2) + (2)(21)(1) + (3)(15)(1)]\ modulo\ 105$

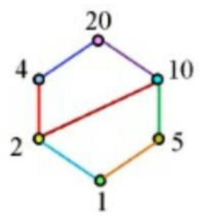
$x \equiv 157\ (mod\ 105)$

$x \equiv 52\ (mod\ 105)$

$\therefore$ $\boxed{x = 52}$ is one solution . General solution is given by, $\boxed{x = 52 + 105\ k}$ where k is any integer

6b) Given L = {1, 2, 4, 5, 10, 20} with divisibility relation. Verify that (L,≤) is a distributive but not complimented Lattice. (6)

Ans. The Hasse Diagram for L is



consider $4 \vee 5 = 20$

$\therefore 2 \wedge (4 \vee 5) = 2 \wedge 20 = 2$

Also,

$2 \wedge 5 = 1$ & $2 \wedge 4 = 2$,

$\therefore (2 \wedge 4) \vee (2 \wedge 5) = 2 \vee 1 = 2$

$\therefore 2 \wedge (4 \vee 5) = (2 \wedge 4) \vee (2 \wedge 5)$

$\because$ Distributive property is satisfied.

Consider $5 \wedge 10 = 5$

$\therefore 4 \vee (5 \wedge 10) = 4 \vee 5 = 20$

Also, $4 \vee 5 = 20$ & $4 \vee 10 = 20$,

$\therefore (4 \vee 5) \wedge (4 \vee 10) = 20 \vee 20 = 20$

$\therefore 4 \vee (5 \wedge 10) = (4 \vee 5) \wedge (4 \vee 10)$

$\therefore$ Distributive property is satisfied.

$\therefore$ (L ,≤) is a distributive Lattice.

(ii)By definition of complement, $a \vee \bar{a} = 1$ and $a \wedge \bar{a} = 0$ i.e. $a \vee \bar{a} = 20$ and $a \wedge \bar{a} = 1$

The complements of elements of set L are

Element	1	2	4	5	10	20
complement	20	--	5	4	--	1

$\therefore$Complement of each element do not exists.

$\therefore$ (L, ≤) is not a complemented Lattice.

6c) Draw a complete graph of 5 vertices. (4)

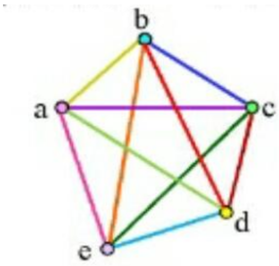
Ans. Definition: In graph theory, a simple graph is a complete graph in which every pair of vertices are adjacent.

A complete graph with 'n' graph vertices is denoted k_n.

Degree of every vertex = (n – 1)

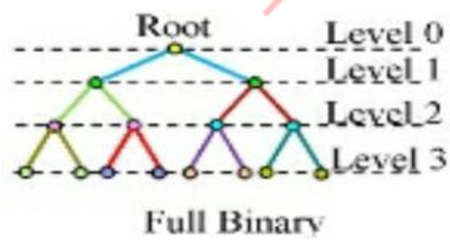
It has $\frac{n\ (n-1)}{2}$ number of edges

A complete graph with '5' vertices is as shown



6d) Give an example of tree. (4)

Ans. Definition: Trees are graphs that do not contain even a single cycle. Every acyclic connected graph is a tree, and vice versa.
The edges of a tree are known as branches.
Elements of trees are called their nodes.
A tree with n vertices has n-1 edges.
A tree in which one vertex (called the root)is distinguished from all the others is called a rooted tree.
An example of a TREE is "Binary Tree".
A tree in which one and only one vertex has a degree two and the remaining vertices are of degree one or three is called a Binary Tree.



A binary tree has three or more vertices.
Since the vertex of degree two is distinct from all other vertices, it serves as a root, and so every binary tree is a rooted tree.